

NAG Toolbox for MATLAB

f08ha

1 Purpose

f08ha computes all the eigenvalues and, optionally, all the eigenvectors of a real n by n symmetric band matrix A of bandwidth $(2k_d + 1)$.

2 Syntax

```
[ab, w, z, info] = f08ha(jobz, uplo, kd, ab, 'n', n)
```

3 Description

The symmetric band matrix A is first reduced to tridiagonal form, using orthogonal similarity transformations, and then the QR algorithm is applied to the tridiagonal matrix to compute the eigenvalues and (optionally) the eigenvectors.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobz** – string

If **jobz** = 'N', compute eigenvalues only.

If **jobz** = 'V', compute eigenvalues and eigenvectors.

Constraint: **jobz** = 'N' or 'V'.

2: **uplo** – string

If **uplo** = 'U', the upper triangular part of A is stored.

If **uplo** = 'L', the lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

3: **kd** – int32 scalar

If **uplo** = 'U', the number of superdiagonals, k_d , of the matrix A .

If **uplo** = 'L', the number of subdiagonals, k_d , of the matrix A .

Constraint: **kd** ≥ 0 .

4: **ab(ldab,*)** – double array

The first dimension of the array **ab** must be at least **kd** + 1

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The upper or lower triangle of the n by n symmetric band matrix A .

The matrix is stored in rows 1 to $k_d + 1$, more precisely,

if **uplo** = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in **ab**($k_d + 1 + i - j, j$) for $\max(1, j - k_d) \leq i \leq j$;
 if **uplo** = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in **ab**($1 + i - j, j$) for $j \leq i \leq \min(n, j + k_d)$.

5.2 Optional Input Parameters

1: **n** – **int32 scalar**

Default: The second dimension of the array **ab**.

n , the order of the matrix A .

Constraint: $n \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldab, ldz, work

5.4 Output Parameters

1: **ab**(ldab,*) – **double array**

The first dimension of the array **ab** must be at least $k_d + 1$

The second dimension of the array must be at least $\max(1, n)$

ab contains values generated during the reduction to tridiagonal form.

The first superdiagonal and the diagonal of the tridiagonal matrix T are returned in **ab** using the same storage format as described above.

2: **w**(*) – **double array**

Note: the dimension of the array **w** must be at least $\max(1, n)$.

If **info** = 0, the eigenvalues in ascending order.

3: **z**(ldz,*) – **double array**

The first dimension, **ldz**, of the array **z** must satisfy

if **jobz** = 'V', $\text{ldz} \geq \max(1, n)$;
 $\text{ldz} \geq 1$ otherwise.

The second dimension of the array must be at least $\max(1, n)$

If **jobz** = 'V', then if **info** = 0, **z** contains the orthonormal eigenvectors of the matrix A , with the i th column of **z** holding the eigenvector associated with **w**(i).

If **jobz** = 'N', **z** is not referenced.

4: **info** – **int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobz**, 2: **uplo**, 3: **n**, 4: **kd**, 5: **ab**, 6: **ldab**, 7: **w**, 8: **z**, 9: **ldz**, 10: **work**, 11: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** = i , the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the *machine precision*. See Section 4.7 of Anderson *et al.* 1999 for further details.

8 Further Comments

The total number of floating-point operations is proportional to n^3 if **jobz** = 'V' and is proportional to $k_d n^2$ otherwise.

The complex analogue of this function is f08hn.

9 Example

```
jobz = 'Vectors';
uplo = 'U';
kd = int32(2);
ab = [0, 0, 3, 4, 5;
      0, 2, 3, 4, 5;
      1, 2, 3, 4, 5];
[abOut, w, z, info] = f08ha(jobz, uplo, kd, ab)
```

abOut =

0	0	3.0000	6.9338	1.5841
0	3.6056	6.9682	-2.3328	-0.2640
1.0000	5.4615	8.9115	2.8591	-3.2322

w =

-3.2474
-2.6633
1.7511
4.1599
14.9997

z =

0.0394	0.6238	0.5635	-0.5165	0.1582
0.5721	-0.2575	-0.3896	-0.5955	0.3161
-0.4372	-0.5900	0.4008	-0.1470	0.5277
-0.4424	0.4308	-0.5581	0.0470	0.5523
0.5332	0.1039	0.2421	0.5956	0.5400

info =

0
